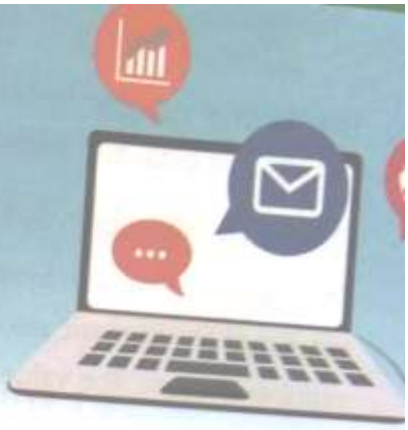
1 NUMBER SYSTEM



Your Aim

to learn about:

- Number system
- Binary to decimal conversion
- Decimal to binary conversion
- Operations on binary numbers

INTRODUCTION

Although the words, data and information are often used interchangeably, but there is an important distinction between the two words. In the strict sense, data consists of the raw numbers that computers organise to produce information.

From early age, we are introduced to the concept of numbers and counting. Toddlers learn at an early age that they can carry two cookies, one in each hand. Kindergarteners start counting by twos and fives. Invariably, we use the decimal number system. Our number system is based on 10, most likely because we have 10 fingers. Let us learn more about the number system.

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- The numerical digits, we use today, i.e, 1, 2 and 3 are based on the Hindu-Arabic numeral system developed over 1000 years ago.
- Different names for the number 0 include zero, nought, naught, nil, zilch and zip.

NUMBER SYSTEM

A number system is simply a method of counting. Many different number systems exist. Consider a clock. Clocks have 24 hours, each composed of 60 minutes. Each minute is in turn composed of 60 seconds. When we time a race, we count in seconds and minutes. Computers, like clocks, have their own number system, the binary system.

The digital computer represents all kinds of data and information in binary numbers. It includes audio, graphics, video, text and numbers. The total number of digits used in a number system is called its **base** or **radix**. Therefore, when someone says that they are working with number system of radix 2, it means base 2, that is, binary number system. The base is written after the number as subscript such as $(512)_{10}$ is wirtten for a number expressed in decimal number system.

Some important number systems are given below:

1. Decimal number system
2. Binary number system
3. Octal number system
4. Hexadecimal number system

The decimal number system is used in general. However, the computers use binary number system. The octal and hexadecimal number systems are also used in the computer.

Decimal Number System

The Decimal Number System consists of ten digits from 0 to 9. These digits can be used to represent any numeric value. The base of decimal number system is 10. It is the most widely used number system. The value represented by individual digit depends on the weight and position of the digit.

Each number in this system consists of digits which are located at different positions. The position of first digit towards left side of the decimal point is 0. The position of second digit towards left side of the decimal point is 1. Similarly, the position of first digit towards right side of decimal point is –1. The position of second digit towards right side of decimal point is –2, and so on.

The value of the number is determined by multiplying the digits with the weight of their position and adding the results. This method is known as **expansion method**. The rightmost digit of the number has the lowest weight. This digit is called **Least Significant Digit** (**LSD**). The leftmost digit of a number has the highest weight. This digit is called **Most Significant Digit** (**MSD**). The digit 7 in the number 724 is the most significant digit and 4 is the least significant digit.

The following table shows Positional weight of decimal number $(724)_{10}$.

	Hundereds	Tens	Units
Weights	10^2	10^1	10^0
Real Value of each digit	7×10^2	2×10^1	4×10^0
Result	700	20	4

Binary Number System

The word binary comes from 'Bi-' meaning two. We see 'bi-' in words such as 'bicycle' (two wheels) or 'binocular' (two eyes). The binary numbers have the base of 2.

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A single binary digit (like '0' or '1') is called a 'bit'. For example 11010 is five bits long. The word bit is made up from the words 'binary digit'.

A computer is a machine made up of transistors, switches and other components. All these electronic components are in two mutually exclusive states, either ON or OFF. The two binary digits represent these two states. Every instruction to the computer is, therefore, converted into 0's and 1's so that it is understood and implemented. This language is therefore known as the machine language.

A binary number is made up of only **0**s and **1**s.

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Name	Size (bits)	Examples
Bit	1	Single digit either 0 or 1
Nibble	4	Group of 4 digits either 0 or 1
Byte	8	Group of 8 digits either 0 or 1
Word	16	Group of 16 digits either 0 or 1

Example of Binary Number:

110100

There is no 2, 3, 4, 5, 6, 7, 8 or 9 in Binary!

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If the last digit of a binary number is 1, the number is odd; if it is 0, the number is even.

For Example: 1101 represents an odd number (13); 10010 represents an even number (18)

Let us first learn how to form binary numbers.

Binary Number System consists of two digits 0 and 1. Its base is 2. Each digit or bit in binary number system can be 0 or 1. A combination of binary numbers may be used to represent different quantities like 1001. The positional value of each digit in binary number is twice the place value or face value of the digit of its right side. The weight of each position is a power of 2. The place value of the digits according to position and weight is as follows:

Position	3	2	1	0	•	−1	−2
Weights	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}

Octal Number System

Octal Number System consists of eight digits from 0 to 7. The base of octal system is 8. Each digit position in this system represents a power of 8. Any digit in this system is always less than 8. Octal number system is used as a shorthand representation of long binary numbers. The number $(841)_8$ is not valid in this number system as 8 is not a valid digit.

Hexadecimal Number System

The Hexadecimal Number System consists of 16 digits from 0 to 9 and A to F. The letters A to F represent decimal numbers from 10 to 15. The base of this number system is 16. Each digit position in hexadecimal system represents a power of 16. The number $(764)_{16}$ is valid hexadecimal number. It is different from $(764)_{10}$ which is seven hundred and sixty four. This number system provides shortcut method to represent long binary numbers.

DECIMAL TO BINARY CONVERSION

To convert a decimal number into a binary number, follow these steps:

Step 1 Divide the decimal number by 2 (the base of the binary number system).

Step 2 Note down the quotient and the remainder.

Step 3 Divide the quotient obtained again by 2 and note down the resulting quotient and remainder.

Step 4 Repeat the procedure till you reach a quotient less than 2.

Step 5 List the last quotient and all the remainders (moving from bottom to top). You have your binary number.

Look at the given examples to understand the conversion better.

Example 1: Convert the decimal number 26, i.e., $(26)_{10}$ to binary.

2	26
2	13 – 0
2	6 – 1
2	3 – 0
	1 – 1

The binary equivalent of $(26)_{10}$ is **11010**

In other words, $(26)_{10} = (11010)_2$

Start listing the last quotient and all remainders from here.

Example 2: Convert the decimal number 64, i.e., $(64)_{10}$ to binary.

2	64
2	32 – 0
2	16 – 0
2	8 – 0
2	4 – 0
2	2 – 0
	1 – 0

$(64)_{10} = (1000000)_2$

BINARY TO DECIMAL CONVERSION

To convert a binary number into a decimal number, follow the following steps:

- Start from the rightmost digit known as the Least Significant Digit (LSD) before the fractional point, and move leftwards.
- While doing so, multiply each digit by 2 raised to a particular power. The powers of 2 start from 0 and increase to 1, 2, and so on as you move leftwards.
- Add up all the resulting products. You have your decimal number.

The following examples will help you to understand the conversion.

Example 1: What is $(1111)_2$ in decimal number?

$= 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 8 + 4 + 2 + 1$

$= 15$

$(1111)_2 = (15)_{10}$

Example 2: Convert $(10111)_2$ to decimal number.

$= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 + 2^0$

$= 16 + 0 + 4 + 2 + 1$

$= 23$

$(10111)_2 = (23)_{10}$

Example 3: Convert 101.101 to decimal number.

Position	2	1	0		−1	−2	−3
Face value	1	0	1	•	1	0	1
Weights	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}

101.101 $= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$= 1 \times 4 + 0 + 1 \times 1 + ½ + 0 + 1/8$

$= 4 + 1 + 0.5 + 0.125$

$= 5.625$

$(101.101)_2 = (5.625)_{10}$

OPERATIONS ON BINARY NUMBERS

Binary Addition

Binary addition is similar to the addition of decimal numbers. When the value of addition exceeds the value 1, say 10 or 11, then 1 is carried over to the left of the current position. The rules for adding two binary digits are given below:

X	Y	X + Y
0	0	0 + 0 = 0
0	1	0 + 1 = 1
1	0	1 + 0 = 1
1	1	1 + 1 = 10 (carry 1)

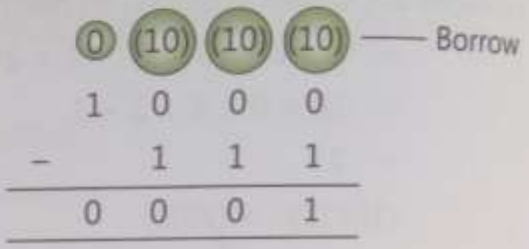
For example let us add the binary numbers $(101111)_2$ and $(10111)_2$.

```
  (1) (1) (1) (1) (1)  ---- Carry bits
   1   0   1   1   1   1
+      1   0   1   1   1
-------------------------
 1   0   0   0   1   1   0
```

Binary Subtraction

In binary subtraction, binary number of lower value is subtracted from the binary number of higher value. The following table explains the subtraction of digit Y from digit X. If Y is greater than X, then 1 is borrowed from the next position. When the binary digit 0 borrows 1 from the next most significant digit, it becomes 10.

X	Y	X – Y
0	0	0 – 0 = 0
0	1	0 – 1 = 1 (borrow 1, so that 10 – 1 = 1)
1	0	1 – 0 = 1
1	1	1 – 1 = 0

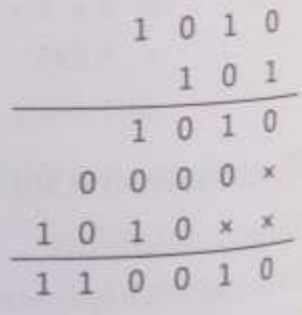


For example let us subtract the binary number $(111)_2$ from $(1000)_2$.

Binary Multiplication

Binary numbers are multiplied in the same manner as decimal numbers. When two binary numbers are multiplied, for each digit of the second number, the product of that digit of the first binary number is calculated. Then it is shifted leftwards so that its rightmost digit lines up with the digit in second number that was used. The sum of all these partial products gives the result. The following rules are followed to multiply two binary numbers.

X	Y	X × Y
0	0	0 × 0 = 0
0	1	0 × 1 = 0
1	0	1 × 0 = 0
1	1	1 × 1 = 1



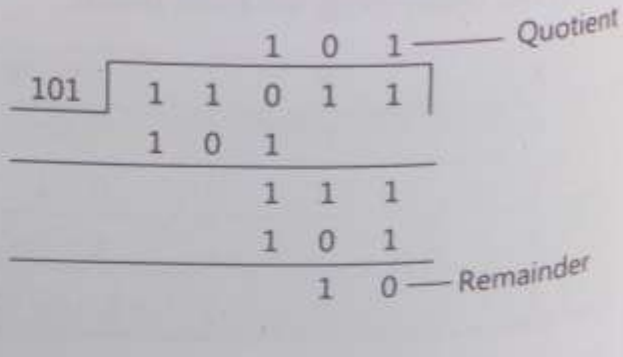
For example, let us multiply $(1010)_2$ by $(101)_2$.

Binary Division

Binary numbers are also divided in a manner similar to decimal numbers. The division is done by the repeated process of subtraction. In binary division, the following rules are followed:

0 / 1 = 0

1 / 1 = 1



For example, let us divide $(11011)_2$ by $(101)_2$.

Reboot

- A number system is simply a manner of counting. Many different number systems exist.
- The Decimal number system consists of ten digits from 0 to 9. These digits can be used to represent any numeric value. The base of decimal number system is 10.
- Computers use binary digits.
- Octal number system is used as a shorthand representation of long binary numbers. The number 6418 is not valid in this number system as 8 is not a valid digit.
- The Hexadecimal number system consists of 16 digits from 0 to 9 and A to F. The letters A to F represent decimal numbers from 10 to 15.

One Touch Learn

A. Tick (✓) the correct option.

1. In a binary number system right most digit before the fractional point is called ..... .
   a. MSD (Most Significant Digit) ☐ c. LSD (Least Significant Digit) ☐
   b. registers ☐ d. None of these ☐

2. Base 2 is another name for the ..... .
   a. Binary number system ☐ c. Hexadecimal number system ☐
   b. Decimal number system ☐ d. None of these ☐

3. ..... are the two symbols present in the binary number system.
   a. 1 and 2 ☐ c. 0 and 1 ☐
   b. 8 and 9 ☐ d. 5 and 6 ☐

4. The decimal number $(345)_{10}$ is equivalent to ..... .
   a. $(101011001)_2$ ☐ c. $(110111001)_2$ ☐
   b. $(111011101)_2$ ☐ d. $(111110001)_2$ ☐

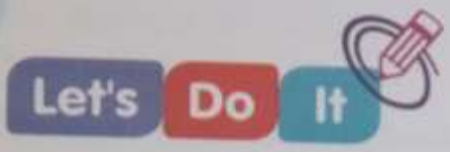
B. Write 'T' for true and 'F' for false. Corect the false statements.

1. A number system is simply a manner of counting. ..... T .....
2. The base of decimal number system is 10. ..... T .....
3. The word binary comes from 'Bi-' meaning two. ..... T .....
4. Octal number system is used as a shorthand representation of long binary numbers. ..... T .....
5. The Hexadecimal number system consists of 16 digits. ..... T .....

C. Fill in the blanks using the words given below.

binary, 0, 2, 8, 10, decimal number

1. In binary subtraction, 1 – 1 equals to 0 .
2. The base of binary number system is 2 .
3. The base of Decimal Number system is 10.
4. In binary addition, 1 + 1 is equal to 10 .
5. Octal number consists of 8 digits.
6. Computer system understands Binary numbers.



A. Short answer type questions.

1. What is an octal number?
2. What is a byte?
3. What is the base in octal number system and hexadecimal number system?

B. Long answer type questions.

1. What is number system? Explain.
2. What are the rules to convert a decimal number into a binary number?
3. Write the rules to multiply two binary numbers.

C. Solve the following.

1. Convert $(10111.011)_2$ into its decimal equivalent.
2. Subtract $(1101)_2$ from $(100110)_2$.
3. Add the following binary numbers.
   a. $(1000)_2$ and $(101)_2$ b. $(1011)_2$ and $(101)_2$ c. $(1001)_2$ and $(101)_2$
   d. $(1111)_2$ and $(1111)_2$ e. $(11001)_2$ and $(10101)_2$
4. Convert the following decimal numbers into binary.
   a. $(39)_{10}$ b. $(72)_{10}$ c. $(128)_{10}$
   d. $(55)_{10}$ e. $(173)_{10}$

A - Short answer type questions –

Qu. 1 What is an octal number?

Answer Octal Number system consists of eight digits from 0 to 7. The base of Octal system is 8. Each digit position in this system represents a power of 8. Any digit in this system is always less than 8.

Qu. 2 What is a byte?

Answer A group of binary digits or bits (usually eight) operated on as a unit. A byte considered as a unit of memory size.

Qu. 3 What is a bit?

Answer A single binary digit (like '0' or '1') is called a 'bit'. For example 11010 is five bits long. The word bit is made up from the words 'binary digit'.

Qu. 4 What is the base in octal number system and hexadecimal number system?

Answer *Octal Number System* base of 8 (0 to 7) and

Hexadecimal Number System base of 16 (0 to 9 and A to F)

B - Long answer type questions –

Qu. 1 What is number system? Explain.

Answer A number system is simply a method of counting. Many different number systems exist. Consider a clock. Clocks have 24 hours, each composed of 60 minutes. Each minute is in turn composed of 60 seconds. When we time a race, we count in seconds and minutes. Computers, like clocks have their own number system, the binary system.
Some important number system are given below -

1. Decimal Number System
2. Binary Number System
3. Octal Number System
4. Hexadecimal Number System

Qu. 2 What are the rules to convert a decimal number into a binary number?

Answer To convert a decimal number into a binary number, follow these steps-

Step 1 – Divide the decimal number by 2 (the base of the binary number system)

Step 2 – Note down the quotient and the remainder.

Step 3 – Divide the quotient obtained again by 2 and note down the resulting quotient and remainder.

Step 4 – Repeat the procedure till you reach a quotient less than 2.

Step 5 – List the last quotient and all the remainders (moving from bottom to top). You have your binary number.

Look at the given examples to understand the conversion better.

Example 1: Convert the decimal number 26, i.e., $(26)_{10}$ to binary.

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2	13 – 0
2	6 – 1
2	3 – 0
	1 – 1

The binary equivalent of $(26)_{10}$ is **11010**
In other words, $(26)_{10} = (11010)_2$

Start listing the last quotient and all remainders from here.

Example 2: Convert the decimal number 64, i.e., $(64)_{10}$ to binary.

2	64
2	32 – 0
2	16 – 0
2	8 – 0
2	4 – 0
2	2 – 0
	1 – 0

$(64)_{10} = (1000000)_2$

Qu. 3 Write the rules to multiply two binary numbers.

Answer Binary numbers are multiplied in the same manner as decimal numbers. When two binary numbers are multiplied, for each digit of the second number, the product of that digit of the first binary number is calculated. Then it is shifted leftwards so that its rightmost digit lines up with the digit in second number that was used. The sum of all these partial products gives the result. The following rules are followed to multiply two binary numbers.

X	Y	X × Y
0	0	0 × 0 = 0
0	1	0 × 1 = 0
1	0	1 × 0 = 0
1	1	1 × 1 = 1

```
  1 0 1 0
    1 0 1
-----------
  1 0 1 0
0 0 0 0 x
1 0 1 0 x x
-----------
1 1 0 0 1 0
```

For example, let us multiply $(1010)_2$ by $(101)_2$.